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# SEISMIC SOIL-STRUCTURE INTERACTION: BENEFICIAL OR DETRIMENTAL?

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# SEISMIC SOIL-STRUCTURE INTERACTION: BENEFICIAL OR DETRIMENTAL?

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The role of soil-structure interaction (SSI) in the seismic response of structures is reexplored using recorded motions and theoretical considerations. *Firstly*, the way current seismic provisions treat SSI effects is briefly discussed. The idealised design spectra of the codes along with the increased fundamental period and effective damping due to SSI lead invariably to reduced forces in the structure. Reality, however, often differs from this view. It is shown that, in certain seismic and soil environments, an increase in the fundamental natural period of a moderately flexible structure due to SSI may have a detrimental effect on the imposed seismic demand. *Secondly*, a widely used structural model for assessing SSI effects on inelastic bridge piers is examined. Using theoretical arguments and rigorous numerical analyses it is shown that indiscriminate use of ductility concepts and geometric relations may lead to erroneous conclusions in the assessment of seismic performance. Numerical examples are presented which highlight critical issues of the problem.

Keywords: Soil-structure interaction, earthquake, ductility, inelastic response, seismic regulations, bridge.

#### 1. Introduction

It is well known that established seismic design methods may involve tenets and practices that are not always accurate. This is especially so in codified approaches which incorporate substantial approximations to provide simple frameworks for design. Several such examples, including cornerstone issues such as the relation between structural strength and ductility, were identified by Priestley (1993) as "myths" and "fallacies" in earthquake engineering.

Seismic soil-structure interaction (SSI) is not free of misconceptions. Despite extensive research over than 30 years in this subject [see pertinent publications by Jennings and Bielak, 1973; Veletsos and Meek, 1974; Bielak, 1975; Roesset, 1980; Luco, 1982; Wolf, 1985; Ciampoli and Pinto, 1995; Gazetas and Mylonakis, 1998, Stewart *et al.*, 1999], there is still controversy regarding the role of SSI in the seismic performance of structures founded on soft soil. In fact, SSI has been traditionally considered to be *beneficial* for seismic response. Neglecting SSI effects is currently being suggested in many seismic codes (ATC-3, NEHRP-97) as a conservative simplification that would supposedly lead to improved safety margins. Apparently, this perception stems from oversimplifications in the nature of seismic demand adopted in code provisions. The most important of these simplifications (with reference to SSI) are: (1) acceleration design spectra that decrease monotonically with increasing structural period; (2) response modification coefficients (i.e. "behaviour factors" used to derive design forces) which are either constant (period-independent) or increase with increasing structural period; (3) foundation impedances derived assuming homogeneous halfspace conditions for the soil, which tend to overpredict the damping of structures on actual soil profiles.

Additional support to the belief of an always beneficial SSI has come from analytical studies of the seismic response of elastoplastic oscillators. Results from several such studies, performed for both fixed-base [Newmark and Hall, 1973; Ridell and Newmark, 1979; Hidalgo and Arias, 1990] and flexibly-supported systems [Ciampoli and Pinto, 1995; Elnashai and McClure, 1996], have shown that the ductility *demand* imposed on an elastoplastic structure tends to *decrease* with increasing elastic structural period. Other analyses, however, [Miranda and Bertero, 1994], based on motions recorded on soft soil deposits, indicate that in certain frequency ranges the trend may reverse that is, ductility demand may *increase* with increasing period. In addition, theoretical studies by Priestley and Park (1987) showed that the additional flexibility of an elastoplastic bridge pier due to the foundation compliance reduces the ductility *capacity* of the system, an apparently detrimental consequence of SSI.

The first objective of the paper is to evaluate the approach seismic regulations propose for assessing SSI effects. This is done in two parts: (a) by examining the effects of SSI on the response of *elastic* single-degree-of-freedom (SDOF) oscillators; and (b) by examining the effects of increase in period due to SSI on the ductility demand imposed on *elastoplastic* SDOF oscillators. The second objective is to evaluate the model of Priestley and Park (1987) for assessing SSI effects in elastoplastic bridge piers. To this end, the model is used to explore the role of SSI on: (i) the ductility *capacity* of the piers; and (ii) the corresponding ductility demand imposed on such systems, if founded on soft soil, during strong earthquake motion. It is noted that the scope of this work is to highlight rather than fully resolve the above issues. The paper focuses on SSI effects in firm nonliquefiable soil, produced by the inertia of the superstructure ("inertial" interaction). Kinematic effects associated with scattering of incoming seismic waves by the foundation (which may also be of importance in certain cases) and SSI effects in liquefiable soil, are not addressed.

# 2. SSI and Seismic Code Spectra

The presence of deformable soil supporting a structure affects its seismic response in many different ways, as illustrated in Fig. 1. Firstly, a flexibly-supported structure has different vibrational characteristics, most notably a longer fundamental period,  $\tilde{T}$ , than the period T of the corresponding rigidly-supported (fixed-base) structure. Secondly, part of the energy of the vibrating flexibly-supported structure is dissipated into the soil through wave radiation (a phenomenon with no counterpart in rigidly-supported structures) and hysteretic action, leading to an effective damping ratio,  $\tilde{\beta}$ , which is usually larger than the damping  $\beta$  of the corresponding fixed-base structure.



Fig. 1. Effect of soil-structure interaction on fundamental matural period and effective damping of a structure on flexible foundation according to NEHRP-97 provisions.



Fig. 2. Reduction in design base shear due to SSI according to NEHRP-97 seismic code.

The seismic design of structures supported on deformable ground must properly account for such an increase in fundamental period and damping. Following early studies by Jennings and Bielak (1973) and Veletsos and Meek (1974), the Applied Technology Council's provisions for the development of seismic regulations (known widely as ATC-3), proposed simple formulae for computing  $\tilde{T}$  and  $\tilde{\beta}$  of structures founded on mat foundation on a homogeneous halfspace. With these two fundamental variables, the engineer can use the appropriate design spectrum to derive the design seismic forces.

With little exception (e.g. NZS4203), seismic codes today use idealized smooth design spectra which attain constant acceleration up to a certain period (of the order of 0.4 s to 1.0 s at most, depending on soil conditions), and thereafter decrease monotonically with period (usually in proportion to  $T^{-1}$  or  $T^{-2/3}$ ). As a consequence, consideration of SSI leads invariably to *smaller* accelerations and stresses in the structure and its foundation.<sup>a</sup> For example, the reduction in base shear according to NEHRP-97 is expressed as (Fig. 2):

$$\Delta V = \left[ C_s(T,\beta) - C_s(\tilde{T},\beta) \left(\frac{\beta}{\tilde{\beta}}\right)^{0.4} \right] W \tag{1}$$

where  $C_s$  is the seismic response coefficient obtained from the spectrum and W is the weight of the structure; the term  $(\beta/\tilde{\beta})^{0.4}$  on the right-hand side of Eq. (1) accounts for the difference in damping between the rigidly- and the flexibly-supported structure. This "beneficial" role of SSI has been essentially turned into a dogma. Thus, frequently in practice dynamic analyses avoid the complication of accounting

<sup>&</sup>lt;sup>a</sup>The increase in period due to SSI leads to higher relative displacements which, in turn, may cause an increase in seismic demand associated with P- $\Delta$  effects. This effect, however, is consided to be of minor importance (NEHRP-97).

for SSI — a supposedly conservative simplification that would lead to improved safety margins. This beneficial effect is recognized in seismic provisions. For example, the NEHRP-97 seismic code states (Commentary, p. 111):

"The (seismic) forces can therefore be evaluated conservatively without the adjustments recommended in Sec. 5.5 (i.e. for SSI effects)."

Since design spectra are derived conservatively, the above statement may indeed hold for a large class of structures and seismic environments. But not always. There is evidence documented in numerous case histories that the perceived beneficial role of SSI is an oversimplification that may lead to unsafe design for both the superstructure and the foundation.

To elucidate this, the ordinates of a conventional design spectrum for soft deep soil, are compared graphically in Fig. 3 against four selected response spectra: Brancea (Bucharest) 1977, Michoacan [Mexico City (SCT)] 1985, Kobe (Fukiai, Takatori) 1995, presented in terms of spectral amplification. Notice that all the recorded spectra attain their maxima at periods exceeding 1.0 s. The large spectral values of some of these records are undoubtedly the result of resonance of the soil deposit with the incoming seismic waves (as in the case with the Mexico City SCT record). Another phenomenon, however, of seismological rather than geotechnical



Fig. 3. Comparison of a typical seismic code design spectrum to actual spectra from catastrophic earthquakes with strong long-period components;  $\beta = 5\%$ .

nature, the "forward fault-rupture directivity" (Somerville, 1998), may be an important contributing factor in the large spectral values at T > 0.50 s in near-fault seismic motions (e.g. in Takatori and Fukiai). As noted by Somerville, an earthquake is a shear dislocation that begins at a point on a fault and spreads outward along the fault at almost the prevailing shear wave velocity. The propagation of fault rupture toward a site at very high velocity causes most of the seismic energy from the rupture to arrive in a single long-period pulse of motion, at the beginning of the recording (Somerville et al., 1997). This pulse is sometimes referred to as "fling". The radiation pattern of the shear dislocation on the fault causes this large pulse of motion to be oriented in the direction perpendicular to the fault, causing the strike-normal peak velocity to be larger than the strike-parallel velocity. The effect of forward rupture directivity on the response spectrum is to increase the spectral values of the horizontal component normal to the fault strike at periods longer than about 0.5 s. Examples of this effect are the Kobe (1995) JMA, Fukiai, Takatori, and Kobe University records; the Northridge (1994) Rinaldi, Newhall, Sylmar Converter, and Sylmar Olive View records; the Landers (1992) Lucerne Valley record, and many others. Figure 4 shows the effects of rupture directivity in the time history and response spectrum of the Rinaldi record of the 1994 Northridge



Fig. 4. Acceleration and velocity time histories for the strike-normal and strike-parallel horizontal components of ground motion, and their 5% damped response spectra, recorded at Rinaldi during the 1994 Northbridge earthquake. Note the pronounced high velocity/long period pulse in the fault-normal component (after Somerville, 1998).

earthquake. Evidently, records with enhanced spectral ordinates at large periods are not rare in nature — whether due to soil or seismological factors.

It is therefore apparent that as a result of soil or seismological factors, an increase in the fundamental period due to SSI may lead to increased response (despite a possible increase in damping), which contradicts the expectation incited by the conventional design spectrum. It is important to note that all three earthquakes presented in Fig. 3 induced damage associated with SSI effects. Mexico earthquake was particularly destructive to 10- to 12-storey buildings founded on soft clay, whose period "increased" from about 1.0 s (under the fictitious assumption of a fixed base) to nearly 2.0 s in reality due to SSI [Resendiz and Roesset, 1985]. The role of SSI on the failure of the 630 m elevated highway section of Hanshin Expressway's Route 3 in Kobe (Fukae section) has also been detrimental and is discussed later on [see also Gazetas and Mylonakis, 1998; Mylonakis *et al.*, 2000]. Evidence of a potentially detrimental role of SSI on the collapse of buildings in the recent Adana-Ceyhan earthquake was presented by Celebi (1998).

It should be noted that due to SSI large increases in the natural period of structures  $(\tilde{T}/T > 1.25)$  are not uncommon in relatively tall yet rigid structures founded on soft soil [Fig. 1; Tazoh *et al.*, 1988; Mylonakis *et al.*, 1997; Stewart *et al.*, 1999]. Therefore, evaluating the consequences of SSI on the seismic behavior of such structures may require careful assessment of *both seismic input and soil conditions*; use of conventional design spectra and generalised/simplified soil profiles in these cases may not reveal the danger of increased seismic demand on the structure.

To further illustrate the above, results from a statistical study performed by the authors using a large set of motions recorded on soft soil are presented. The set of motions consists of 24 actual records (Appendix I); this is an extended version of the set used by Miranda (1991). The average acceleration spectrum obtained from these motions is presented in Fig. 5, in terms of spectral amplification. The structural period is presented in three different ways: (i) actual period T; (ii) normalised period  $T/T_g$  [ $T_g$  = "effective" ground period, defined as the period where the 5% velocity spectrum attains its maximum [Miranda and Bertero, 1994]; (iii) normalised period  $T/T_a$  [ $T_a$  = period where acceleration spectrum attains its maximum.] It is seen that with the actual period, the resulting average spectrum has a flat shape (analogous to that used in current seismic codes) which has little resemblance to an actual spectrum. The reason for this unrealistic shape is because the spectra of motions recorded on soft soil attain their maxima at different, well separated periods and, thereby, averaging them eliminates the peaks causing this effect. In contrast, with the normalised periods  $T/T_g$  and  $T/T_a$  the average spectrum exhibits a characteristic peak close (but not exactly equal) to 1, which reproduces the trends observed in actual spectra. It is well known that the issue of determining a characteristic "design" period (i.e.  $T_g$  or  $T_a$ ) for a given site is controversial and, hence, it has not been incorporated in seismic codes. Nevertheless, it is clear that current provisions treat seismic demand in soft soils in a nonrational way, and may provide designers with misleading information on the significance of SSI effects.



Fig. 5. Average acceleration spectra based on 24 actual motions recorded on soft soil. The periods are either normalised before averaging with: (a) period of peak spectral acceleration  $(T_a)$ ; (b) period of peak spectral velocity  $(T_g)$ ; or (c) average without any normalisation;  $\beta = 5\%$ .

As a final remark, it should be mentioned that some new methods in seismic design may modify some of the existing perceptions associated with SSI. For example, recently developed procedures for displacement-based design utilise *displacement* (instead of acceleration) spectra, developed directly from strong motion records [Bommer and Elnashai, 1999; Tolis and Faccioli, 1999]. Contrary to acceleration spectra, displacement spectra exhibit increasing trends over large ranges of periods and, thereby, SSI effects will not appear as being invariably beneficial for seismic response. More details on these new methods can be found in [Kowalski *et al.*, 1994; Calvi and Kingsley, 1995; and Bommer and Elsnashai, 1999].

#### 2.1. Effect of period lengthening on inelastic response

The foregoing discussion was based on the assumption that the response of the structure is linearly viscoelastic. However, during strong earthquake shaking a structure may exhaust its elastic strength and deform beyond its yielding point (i.e. inelastically) without collapsing. Accordingly, engineers design structures with strength which is only a fraction of that required to prevent yielding (elastic force demand), provided that the displacement imposed to the structure by the earthquake (displacement demand) is smaller than the ultimate displacement the structure can sustain (displacement capacity). The foregoing can be put in a dimensionless form in terms of the following well-known parameters: (i) ductility demand  $\mu$  (= displacement demand/yielding displacement); (ii) response modification factor R (= elastic force demand/yielding strength).

Yielding of elastoplastic structures has some distinct similarities to the effects of SSI on elastic structures. Both phenomena result to an increase in the "effective" natural period and damping of the structure. In this section our focus is on the effect of the increased *initial* period of the structure on ductility demand. To this end, we examine the relation between ductility demand  $\mu$  and strength reduction factor R as functions of structural period.

In a seminal work, Newmark and Hall (1973) proposed two approximate relationships between  $\mu$  and R. Using a limited number of recorded motions available at that time, they observed that: (1) in the moderately-long and long period ranges, an elastic and an inelastic oscillator of the same initial period have approximately the same maximum relative displacement ("equal displacement rule"); (2) in the moderately-short period range, the energy defined by the area under a monotonic force-displacement diagram is approximately the same for an elastic and an inelastic oscillator ("equal energy rule"). Based on these assumptions it is a simple matter to show that  $\mu$  and R can be related as

$$\mu = \begin{cases} \frac{(R^2 + 1)}{2}, & \text{moderately - short periods} \\ R, & \text{moderately - long to long periods}. \end{cases}$$
(2)  
(3)

Values for the corresponding period ranges are given in Newmark and Hall (1973) and in Miranda and Bertero (1994).

In the limiting case of a very stiff elastoplastic oscillator,  $T \rightarrow 0$ , and its yielding displacement  $u_y$  is practically zero. If the system has less strength than that required to remain elastic during shaking (R > 1), the ductility demand (computed by dividing the finite displacement response of system by its zero yield displacement) will be of infinite magnitude

$$\mu(T \to 0, \ R > 1) \to \infty. \tag{4}$$

On the other hand, for a very flexible oscillator,  $T \to \infty$ , and the maximum relative displacement will be equal to the peak ground displacement regardless of yielding strength. This leads to the well-known result

$$\mu(T \to \infty, R) = R.$$
<sup>(5)</sup>

Note that contrary to Eqs. (2) and (3) which are approximate, the asymptotic relations (4) and (5) are exact. [In fact, Eq. (3) is an extrapolation of Eq. (5) in the moderately-long period range.]

The trend incited by Eqs. (2) to (5) is clear: For a given R, the ductility demand  $\mu$  will decrease with increasing structural period [i.e. from infinity at zero period, to  $(R^2+1)/2$  at moderately long periods, to R at long periods]. Conversely, for a given "target" ductility  $\mu$  the associated response modification factor R will increase with increasing period. This increase in R implies that the yielding strength required to achieve the prespecified target ductility will tend to decrease with increasing period, and, accordingly, the role of SSI will be beneficial.

With relatively few exceptions (e.g. NZS4203, Caltrans 1990) period-dependent strength reduction factors have not been widely incorporated in seismic codes. [This is apparently because such period-dependent factors are difficult to be embodied into multimode dynamic analyses.] The work of Newmark and Hall (1973) has greatly influenced the development of modern seismic regulations; yet this work further strengthens the belief of an always-beneficial role of SSI.

Although several subsequent studies (based primarily on artificial motions or motions recorded on rock sites) have more or less confirmed the foregoing trends, analytical results based on motions recorded on soft soils trends (see review article by Miranda and Bertero 1994), are in contradiction with the results of Newmark and Hall. Miranda (1991) analysed a large set of ground motions recorded on a wide range of soil conditions and computed strength reduction factors for a set of prespecified ductility demands  $\mu$ . An important finding of his work is that in soft soils (in which SSI effects are typically most pronounced), an increase in structural period may increase the imposed ductility demand. To elucidate this, the expression fitted by Miranda and Bertero (1994) to the mean strength reduction factors for soft soil conditions is illustrated in Fig. 6, plotted in terms of ductility demand  $\mu$  versus structural period. The period is normalised by the predominant period  $T_q$ of the record. Also plotted in the graph are the generic expressions of Newmark and Hall (1973) [(Eqs. (2) to (5)], normalised using  $T_g = 1$  s. For periods larger than about 1.2  $T_g$ , the ductility demand becomes an *increasing* function of period, which contradicts to the trends suggested by Newmark and Hall (1973). Note that the increasing trend becomes stronger-for-weaker oscillators-(i.e. for higher R-values). Similar trends have been presented by Takada et al. (1988) and Nassar and Krawinkler (1991).



Fig. 6. Ductility demand vs. dimensionless structural period. Comparison of Newmark and Hall (1973) with Miranda (1993), for three different levels of force reduction factors. Note the increasing trend at long periods;  $\beta = 5\%$ .



Fig. 7. Comparison of ductility demands imposed by the Fukiai and Takatori records (Kobe 1995) with those obtained from the theory of Newmark and Hall (1973). Note the strong fluctuations of  $\mu$  at long periods;  $\beta = 5\%$ .

Miranda's curves in Fig. 6 represent the statistical average of ductility demands derived from the analysis of a large set of records on soft soil. Apparently, in the case of a single earthquake record, the ductility demand may fluctuate with period much more strongly. This is shown in Fig. 7, in which ductility demands calculated from the Kobe Fukiai and Takatori motions are plotted for three different R factors. The strong fluctuation (especially for R = 4) of  $\mu$  with period is evident.

The foregoing discussion considered structures that were perfectly fixed at the base. The effects of SSI could only be studied indirectly (i.e. through the increase in natural period). A more accurate study of the inelastic response of *flexibly-supported* structures is presented below. It will be shown that, in this case, the interpretation of ductility coefficients may involve pitfalls.

#### 3. Ductility in Flexibly-Supported Structures

To assess the effects of soil flexibility on the inelastic response of structures (particularly bridges) engineers have been using the simple structural idealization of Fig. 8: a single bridge pier connected to the deck monolithically (or through bearings), and subjected to a transverse seismic excitation [Priestley and Park, 1987; Ciampoli and Pinto, 1995]. Elastoplastic bilinear behavior is usually considered for the pier, while the soil-foundation is modelled with translational and rotational springs. Moment-free (cantilever) conditions at the deck are often (but not necessarily) assumed. A simple approach has been proposed for evaluating the effects of SSI on the seismic performance of the inelastic system, by subjecting the bridge pseudostatically to a lateral load.



Fig. 8. The model used to investigate the significance of SSI in the inelastic seismic performance of cantilever bridge piers.

The lateral displacement of the deck relative to the free-field soil,  $\overline{U}$ , can be decomposed as [Priestley and Park, 1987]:

$$U = \Delta_f + \Theta_f H + \Delta_y + \Delta_p \tag{6}$$

in which:

- $\Delta_f$  and  $\Theta_f \times H$  are rigid body displacements of the deck due to the swaying  $(\Delta_f)$  and rocking  $(\Theta_f)$  of the foundation, respectively
- $\Delta_y$  and  $\Delta_p$  represent the yield and plastic displacement of the pier, respectively. [Presence of bearings is not considered for simplicity]
- $\Delta_y = F_y/K_c$  in which  $F_y$  is the yield shear force and  $K_c$  (~ 3  $E_c I_c/H^3$ ) the stiffness of the column
- $\Delta_p$  is the plastic component of deck displacement due to the yielding of pier, which is concentrated at the base of the column ("plastic hinge").

If the column were fixed at its base, Eq. (6) would simplify to:

$$U = \Delta_y + \Delta_p \tag{7}$$

and dividing by  $\Delta_y$  would yield the displacement ductility factor of the column

$$\mu_c = \frac{\Delta_y + \Delta_p}{\Delta_y} \,. \tag{8}$$

For the flexibly-supported system, the yielding displacement,  $\bar{U}_y$ , of the bridge is obtained by setting  $\Delta_p = 0$  to Eq. (6):

$$\tilde{U}_{y} = \Delta_{f} + \Theta_{f} H + \Delta_{y} \,. \tag{9}$$

The ratio  $\tilde{U}/\tilde{U}_y$  defines the socalled "global" or "system" displacement ductility factor of the bridge-foundation system:

$$\mu_s = \frac{U}{\tilde{U}_y} = \frac{\Delta_f + \Theta_f H + \Delta_y + \Delta_p}{\Delta_f + \Theta_f H + \Delta_y} \,. \tag{10}$$

Dividing by  $\Delta_{y}$  yields the dimensionless expression between  $\mu_{s}$  and  $\mu_{c}$ :

$$\mu_s = \frac{c + \mu_c}{c + 1} \tag{11}$$

in which

$$c = (\Delta_f + \Theta_f H) / \Delta_y \tag{12}$$

is a dimensionless coefficient expressing the foundation to structure displacement.

Equations (9) and (10) implicitly assume that the response of the foundation,  $(\Delta_f + \Theta_f \times H)$ , is the same in both yielding and ultimate conditions. This assumption holds for an *perfectly elastic plastic* pier supported on a foundation with higher yielding strength than the column. For a pier with bilinear behavior, Eqs. (10) and (11) should be replaced by

$$\mu_s = \frac{\bar{U}}{\Delta_y (1+c)} \,. \tag{13}$$

Equation (11) is illustrated in Fig. 9 in which  $\mu_s$  is plotted as a function of  $\mu_c$  for different values of the flexibility coefficient c. For c = 0 (a structure fixed at its base), the values of the two factors coincide ( $\mu_s = \mu_c$ ). For c > 0, however,  $\mu_s$  is always smaller than  $\mu_c$ , decreasing monotonically with increasing c. In fact, in



Fig. 9. Relation between pier ductility  $\mu_c$  and system ductility  $\mu_s$  [Eq. (11)] for the bridge model of Fig. 8 (elastic-perfectly plastic pier).

the limiting case of  $c \to \infty$  (an infinitely-flexible foundation or an absolutely rigid structure), the "system" ductility  $\mu_s$  is 1 regardless of the value of  $\mu_c$ .

The above trends have been widely interpreted in the following way [Priestley and Park, 1987; Ciampoli and Pinto, 1995]: Given a ductility capacity  $\mu_c$  of the column ( $\mu_c > 1$ ), the ductility capacity  $\mu_s$  of the SSI system associated with a flexibility ratio c > 0 is lower than it would be for a fixed-base cantilever (with c = 0). As an example, for the typical values  $\mu_c = 4$  (a well-designed column) and c = 1 (a moderately soft soil),  $\mu_s$  is equal to only 2.5, i.e. only 62% of the  $\mu_c$  value.

On the other hand, to achieve a certain ductility capacity for the system, say  $\mu_s = 4$ , the ductility capacity of the column for c = 1 should, according to Eq. (11), be  $\mu_c = 7$ , which may require a substantial increase in deformation. This implies that the additional flexibility due to the foundation compliance reduces the ductility capacity of the system [Priestley and Park, 1987; Ciampoli and Pinto, 1995]. As a straightforward extension to the above statement, one may conclude that soil-structure interaction has a detrimental effect on the inelastic performance of a bridge-foundation system by reducing its ductility capacity.<sup>b</sup> This is in apparent contradiction with the "beneficial" role of SSI discussed earlier. Although evidence for a detrimental role of SSI has already been discussed (and additional such evidence will be presented later on), it will be shown here that drawing such a conclusion using Eq. (11) is incorrect. This is done using two different approaches.

Firstly, consider a counterexample: Suppose that we are interested in the ductility demand (i.e. instead of capacity) imposed to the system by a transient dynamic load. To calculate the ductility demand one must solve the nonlinear equation of motion of the system to determine the peak plastic displacement  $\Delta_p$  (as, for instance, done by Ciampoli and Pinto, 1995). For any value of  $\Delta_p$ , however, Eq. (11) will yield smaller values than Eq. (8) due to the presence of the additional positive number c in both the numerator and denominator (this is exactly what we observe in Fig. 9). Thus, the ductility demand imposed on the SSI system will be smaller than that of a fixed-base system with the same vibrational characteristics and, thereby, SSI will apparently have a *beneficial* role to the system's performance — exactly the opposite to the first interpretation.

The apparent paradox stems from the fact that Eq. (11) is a *kinematic* expression which does not distinguish between capacity and demand; it tends to reduce both ductilities and provides no specific trend on the effect of SSI on the inelastic performance of the system.

The second argument against the validity of  $\mu_s$  as performance indicator is the presence of rigid body displacements (due to the foundation translation and rotation) which are not associated with strain in the pier. In fact, the addition of these displacements in both the numerator and denominator of Eq. (10) is the only

<sup>&</sup>lt;sup>b</sup>Strictly speaking, the changes in both capacity and demand should be considered to conclude whether SSI's role is beneficial or detrimental. Nevertheless, the reduction in ductility capacity suggested by Eq. (11) is obviously detrimental.

reason for the systematic drop in that ratio. This implies that the ductility ratio  $\mu_s$  [expressed through Eqs. (10)-(12)] is not a measure of the distress of the pier, as correctly pointed out by Ciampoli and Pinto (1995). For example, additional rigid body motions could be introduced to the analysis by, say, rotating the reference system during the response. This would reduce  $\mu_s$ , but without having any physical connection to the actual problem. As another example, one may introduce to Eq. (10) the seismic ground displacement. Incorporation of this additional displacement would better reflect the absolute motion of the system, and further reduce  $\mu_s$ . As an extreme case, one may consider the translation due to the motion of the earth; the addition of such a huge displacement to both the numerator and denominator of Eq. (10) will make  $\mu_s$  equal to 1 (implying response without damage) even for a system that has failed!

Finally, it is important to note that  $\mu_s$  in Eq. (10) was derived by examining just the static deflection of the system, i.e. without using time history analysis or any "dynamic" reasoning. In contrast, it is well known that seismic SSI effects are influenced (if not governed) by dynamic phenomena such as resonance and deresonance which cannot be captured by purely static or geometric considerations.

#### 4. Inelastic SSI Analyses

To further investigate the role of SSI on the inelastic performance of bridge piers, nonlinear inelastic analyses were carried out using the model of Fig. 8. Both column and system ductilities were obtained using different oscillators and ground excitations, and results were compared with corresponding demands for fixed-base conditions. A similar investigation has been performed by Ciampoli and Pinto (1995). However, there are some differences between the two studies. While the foregoing study was based on a set of *artificial* ground motions matching the EC-8 (1994) spectrum for intermediate-type soils, the present study uses exclusively *actual* motions recorded on soft soils. In addition, a two-degree-of-freedom system is adopted here (as opposed to a single-degree-of-freedom system in the earlier work), to better represent the dynamic response of the footing. In the present analyses, bilinear elastoplastic behavior is considered for the pier with post-yielding stiffness equaling 10% of the elastic stiffness. A footing mass equal to 20% of the deck mass and a Rayleigh damping equal to 5% of critical in the two elastic modes of the system were considered in all analyses.

Figure 10 presents column ductility demands obtained using the Bucharest (1977) motion. The results are plotted as function of the fixed structural period computed for four different foundation-to-structural flexibility ratios: c = 0 (which corresponds to fixed-base conditions), 0.25, 0.5, and 1. [Note that with c = 1 the period of the flexibly-supported system is  $(1 + c)^{1/2} \approx 1.4$  times higher that that of the corresponding rigid-base system.] In the period range between about 0.5 to 1.5 s, the curves for c > 0 plot above that for c = 0 which implies that SSI increases the ductility demand in the pier. For example, in the particular case T = 0.6 s,



Fig. 10. Effect of SSI on the ductility demand of a bridge pier subjected to the Bucharest Brancea (1977) N-S motion; R = 2.



Fig. 11. Effect of SSI on the ductility demand of a bridge pier subjected to the Mexico City SCT Michoacan (1985) E-W motion; R = 2.

c = 1 ductility increases from 3.5 to about 5.5 without (c = 0) and with (c = 1) SSI, respectively. In smaller periods the increase in  $\mu_c$  is less significant, while at longer periods SSI tends to reduce ductility demand.

Figure 11 refers to the Mexico City SCT (1985) record and R = 2. In this case the effects of SSI are somewhat less significant than in the previous graph. Yet, the tendency for increase in ductility due to SSI is evident with the curves for c > 0plotting above that of c = 0 for periods between 0.70 and 2 s. Incidentally, it should be mentioned that most of the damage caused by this earthquake concentrated in buildings with fundamental fixed-base periods varying from about 0.9 to 1.3 s, which coincides with the region of the maxima of this graph. An interesting case is presented in Fig. 12 referring to the Kobe (1995) Fukiai record. A substantial increase in ductility due to SSI is observed at periods between about 0.5 and 1 s. For example, with a fixed-base period of 0.6 s and c = 1, the ductility demand increases from 2.2 for the fixed-base pier (c = 0) to more than 5 for the flexibly supported system. Similar trends (although not as clear as in Fig. 12) are shown in Fig. 13 for the nearby Kobe (1995) Takatori record. It is important to mention here that the 18 piers (a 630 m segment) at Fukae section of the elevated Hanshin Expressway that failed spectacularly in that earthquake,



Fig. 12. Effect of SSI on the ductility demand of a bridge pier subjected to the Kobe (1995) Fukiai E-W motion; R = 2.



Fig. 13. Effect of SSI on the ductility demand of a bridge pier subjected to the Kobe (1995) Takatori E-W motion; R = 2.

had a fixed-base natural period of about 0.6 s (see Fig. 12), located at a site having similar soil conditions and located similarly with respect to the fault zone as the Fukiai and Takatori sites. The role of SSI on the collapse of that structure was perhaps more significant than originally suspected. More details on this failure are given in Gazetas and Mylonakis (1998), Anastasopoulos (1999), and Mylonakis *et al.* (2000).

Figures 14 and 15 present system ductility demands Eq. (13) obtained from the Bucharest and Mexico City records. It is apparent that the use of system ductil-



Fig. 14. Effect of SSI on the system ductility demand of a bridge pier subjected to the Bucharest (1977) N-S motion; R = 2. Note the reduced values as compared to Fig. 10.



Fig. 15. Effect of SSI on the system ductility demand of a bridge pier subjected to the 1985 Mexico City SCT N-S motion; R = 2. Note the reduction in ductility as compared to Fig. 11.

ity completely obscures the detrimental role of SSI observed in Figs. 10 and 11. Additional insight is provided in the numerical example below.

#### 5. Numerical Example

The simple bridge model of Fig. 8 having mass m = 350 Mg, elastic pier stiffness  $K_c = 48\,000$  kN/m and height H = 6 m, is subjected to the Kobe Fukiai record (Fig. 3) having a peak ground acceleration of 0.80 g. The pier is assumed to be elastoplastic with 10% post-yielding stiffness, designed with a seismic coefficient  $C_s = C_y = 0.50$  which is considered representative of its actual yielding strength. For simplicity, the foundation translation is ignored ( $\Delta_f = 0, K_x = \infty$ ); the rocking stiffness of the footing is  $K_R = 2 \times 10^6$  kN m (this would be the stiffness of a group of  $4 \times 5$  closely-spaced piles embedded in a soil layer with average  $V_s$  of about 200 m/s). For the sake of this demonstration, the damping ratio of the fixed-base and the flexibly-supported system are assumed to be 5% of critical.

# 5.1. Fixed-base system

Period:  $T = 2\pi \sqrt{m/K_c} \simeq 0.54$  s. Displacement of the column at yield:  $\Delta_y = m g C_y/K_c \simeq 3.6$  cm. The acceleration response of the bridge is calculated from the spectrum of Fig. 3 as:

$$S_a(T) = 1.59 \times 0.80 \text{ g} = 1.27 \text{ g}.$$
 (14)

Using a nonlinear inelastic analysis the displacement of the column is obtained as:

$$\Delta = \Delta_y + \Delta_p = 13.0 \text{ cm} \tag{15}$$

from which the ductility demand is computed as

$$\mu_c = \frac{\Delta}{\Delta_y} = \frac{13}{3.6} \approx 3.6 \,. \tag{16}$$

Note that if we had applied the equal displacement rule we would have obtained the value (Eqs. (3) and (1))

$$\mu_c = R = S_a(T) / (C_y g) = 1.27 / 0.5 = 2.5$$
(17)

which underestimates the actual ductility demand. Incidentally, applying the equal energy rule would have yielded the almost exact value  $\mu_c = (R^2 + 1)/2 = (2.5^2 + 1)/2 = 3.63$ .

#### 5.2. Flexibly-supported system

The flexibility coefficient c in this case is given by:

$$c = \frac{K_c H^2}{K_r} = \frac{48\,000 \times 6^2}{2 \times 10^6} \cong 0.86\,. \tag{18}$$

The effective stiffness of the bridge, including SSI effects, is:

$$\bar{K} = \frac{K_c}{1+c} = \frac{48\,000}{1+0.86} \cong 25\,800 \text{ kN/m}$$
(19)

providing a modified period  $\overline{T}$ 

$$\bar{T} = T\sqrt{1+c} \cong 0.74 \text{ s}. \tag{20}$$

Ignoring, for this demonstration and as a first approximation the change in foundation input motion, leads to (Fig. 3):

$$S_a = 2.10 \times 0.80 \text{ g} = 1.68 \text{ g}.$$
 (21)

Note that the above acceleration response [as well as that of Eq. (14)] is fictitious since it can develop only if the structure is perfectly viscoelastic.

Applying a nonlinear inelastic analysis, the peak displacement of the SSI system is determined as:

$$\tilde{U} \cong 29 \text{ cm}. \tag{22}$$

This value includes the rigid-body motion induced by the foundation rotation:

$$\Theta_f H = c[\Delta_y + \alpha (\tilde{U} - c\Delta_y)] = 5.3 \text{ cm}.$$
(23)

Subtracting the above displacement from  $\tilde{U}$ , the ductility demand in the column is obtained as:

$$\mu_c = \frac{\bar{U} - \Theta_f H}{\Delta_v} = \frac{29 - 5.3}{3.6} \cong 6.6 \tag{24}$$

which is much higher than the corresponding demand for the fixed-base system  $(\mu_c = 3.6)$  — a rather anticipated increase in view of the substantial raise in displacement demand (23.7 vs. 13 cm in the two cases, respectively). In contrast, applying Eq. (13) would have yielded a "global" ductility value:

$$\mu_s = \frac{\tilde{U}}{\Delta_y(1+c)} = \frac{29}{3.6(1+0.86)} \cong 4.3.$$
 (25)

This is barely higher than the ductility demand for the rigidly supported structure  $(\mu_c = 3.6)$ , and does not reflect the actual ductility demand in the pier.

#### 6. Conclusions

The main conclusions of this study are:

(1) By comparing conventional code design spectra to actual response spectra, it was shown that an increase in fundamental natural period of a structure due to SSI does not necessarily lead to smaller response, and that the prevailing view in structural engineering of the always-beneficial role of SSI, is an oversimplification which may lead to unsafe design.

- (2) Averaging response spectra of motions recorded on soft soil without proper normalisation of periods may lead to errors.
- (3) Ductility demand in fixed-base structures is not necessarily a decreasing function of structural period, as suggested by traditional design procedures. Analysis of motions recorded on soft soils have shown increasing trends at periods higher than the predominant period of the motions.
- (4) Soil-structure interaction in inelastic bridge piers supported on deformable soil may cause significant increases in ductility demand in the piers, depending on the characteristics of the motion and the structure. However, inappropriate generalization of ductility concepts and geometric considerations may lead to the wrong direction when assessing the seismic performance of such structures.

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## Appendix A

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Station Name	Event/ Date	Geology	Magni- tude (M,)	Epicentral Distance (km)	Direction	PGA (g)	PGV (m/s)
Bucharest Research Inst. Basem. Bldg	Romania 3/4/77	Soft Soil	7.2	149	NS EW	0.21	0.75 0.33
Secretaria Communi- cation & Transport.		Soft Clay		376	NS EW	0.10 0.17	0.39 0.61
Central De Abastos Oficina	Michoacan 9/19/85	Soft Clay	8.1	384	NS EW	0.08 0.07	0.42 0.35
Central De Abastos Frigorifico		Soft Clay		384	NS EW	0.09 0.08	0.35 0.25
Oakland Outer Harbor Wharf		Bay Mud		98	N35E S85W	0.28 0.27	0.42 0.41
Foster City Redwood Shores	Loma Prieta 10/17/89	Bay Mud	7.1	68	NS EW	0.26 0.28	0.32 0.45

Table A1. Selected ground motions recorded at soft soil sites (modified after Miranda, 1991).

Station Name	Event/ Date	Geology	Magni- tude (M <sub>s</sub> )	Epicentral Distance (km)	Direction	PGA (g)	PGV (m/s)
San Francisco 18-storey Commercial Bldg		Fill Over Bay Mud		99	N80E N10W	0.13 0.16	0.17 0.16
Emeryville Free Field South	Loma Prieta 10/17/89	Bay Mud	7.1	97	N10W S80W	0.21 0.26	0.21 0.21
Treasure Island Naval Base		Fill		98	EW NS	0.16 0.10	0.16 0.33
San Francisco International Airport		Bay Mud		79	EW NS	0.33 0.23	0.29 0.26
Colonia Roma	Acapulco 4/25/89	Soft Clay	6.9	_	N90W S00E	0.06 0.05	0.12 0.11
Kobe Fukiai	Kobe 1/17/95	Soft Alluvium	7.2	19 <sup>†</sup>	EW	0.80	1.30
Kobe Takatori		Soft Alluvium		10†	NS _	0.65	1.55

Table A1. (Continued)

<sup>†</sup> near-field records

# Appendix B

# LIST OF SYMBOLS

- $\beta$  damping ratio, damping ratio of a rigidly-supported structure.
- $\tilde{\beta}$  damping ratio of flexibly-supported structure.
- c dimensionless coefficient expressing foundation to structure flexibility ratio.
- $C_s$  seismic response coefficient.
- $\Delta_f$  foundation displacement.
- $\Delta_p$  plastic structural displacement.
- $\mu$  ductility demand.
- $\Delta V$  reduction in base shear.
- $\Delta_y$  yield displacement.
- $E_c I_c$  column flexural rigidity.
- $F_y$  yield shear force.
- g acceleration of gravity.
- H column height.

- $\tilde{K}$  effective stiffness of flexibly-supported system.
- $K_c$  column stiffness.
- $K_R$  rocking stiffness of footing.
- M bridge mass.
- $\mu_c$  column displacement ductility factor.
- $\mu_s$  system displacement ductility factor.
- $\Theta_f$  foundation rotation.
- R response modification factor.
- $S_a$  spectral acceleration.
- T structural natural period, natural period of rigidly-supported structure.
- $\tilde{T}$  natural period of flexibly-supported structure.
- $T_a$  "effective" earthquake period (where 5%-damped acceleration spectrum attains its maximum).
- $T_g$  "effective" earthquake period (where 5%-damped velocity spectrum attains its maximum).
- $u_y$  yielding displacement.
- $\tilde{U}$  lateral displacement of flexibly-supported structure, relative to free-field soil.
- $\tilde{U}_y$  lateral displacement at first yield of flexibly-supported structure.
- W weight of structure.

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